TRANSIENT MOTION OF A LIQUID TO A BOREHOLE IN A DEFORMABLE FRACTURED COLLECTOR

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Transient filtration in a deformable fracture collector is described by nonlinear differential equations of parabolic type

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{rf(P)\frac{\partial P}{\partial r}\right\}=\frac{1}{\varkappa_{T0}}\frac{\partial P}{\partial t},\qquad \varkappa_{T0}=\frac{k_{T0}}{\mu\beta_T^*}\tag{1}$$

in which $\varkappa_{T0}$ is the coefficient of pressure permeability, β_T^* is the coefficient of elastic capacity, $f(P)$ is the pressure function, and k_{T0} is the permeability for $P = P_0$.

As for pressure recovery we seek a solution to (1) by the small-parameter method [1, 2]. We assume that the borehole is put into exploitation at a constant volume flow rate. Let

$$f(P)=[1-\beta(P_0-P)]^3,$$

in which β is a coefficient dependent on the fracturing and the elastic properties [3]. We put (1) as

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\Phi^3\frac{\partial\Phi}{\partial r}\right\}=\frac{1}{\varkappa_{T0}}\frac{\partial\Phi}{\partial t},\quad \Phi=1-\beta(P_0-P).\tag{2}$$

We assume that the flow rate remains constant after the start, i.e., that we have at the wall

$$Q=\frac{2\pi hk_{T0}}{\mu}\left\{r[1-\beta(P_0-P)]^3\frac{\partial P}{\partial r}\right\}_{r=r_c\to0},\tag{3}$$

in which h is the bed thickness and μ is the viscosity of the liquid. We assume that initially the pressure is everywhere constant at

$$P(r,0)=P_0>0\quad\text{or}\quad\Phi(r,0)=\Phi_0>0.\tag{4}$$

For an unbounded fractured bed we have also that

$$P(\infty,t)=P_0>0\quad\text{or}\quad\Phi(\infty,t)=\Phi_0>0.\tag{5}$$

We represent the solution as an infinite series of functions Φ

$$\Phi^4(r,t)=\Phi_0^4+\Omega\Phi_1(r,t)+\Omega^2\Phi_2(r,t)+\Omega^3\Phi_3(r,t)+\ldots,\tag{6}$$

in which $\Phi_1(r,t)$, $\Phi_2(r,t)$, $\Phi_3(r,t)$, ... are to be determined. It follows [4] from (6) that

$$\frac{\Phi(r,t)}{\Phi_0}=\left(1+\frac{\Omega}{\Phi_0^4}\Phi_1(r,t)+\frac{\Omega^2}{\Phi_0^4}\Phi_2(r,t)+\frac{\Omega^3}{\Phi_0^4}\Phi_3(r,t)+\ldots\right)^{1/4}=1+\frac14\frac{\Omega}{\Phi_0^4}\Phi_1(r,t)+$$

$$+\frac14\frac{\Omega^2}{\Phi_0^4}\left[\Phi_2(r,t)-\frac38\frac{\Phi_1^2(r,t)}{\Phi_0^4}\right]+\frac14\frac{\Omega^3}{\Phi_0^3}\times\left[\Phi_3(r,t)-\frac{\Phi_1(r,t)\Phi_2(r,t)}{4\Phi_0^4}+\frac{7}{32\Phi_0^8}\Phi_1^3(r,t)\right]+\cdots$$

$$(\Omega=\mu\beta Q/\pi hk_{T0}\quad(\Omega\ll1)).\tag{7}$$

Substitution of (6) and (7) into (2) gives us the following chain of differential equations:

$$\frac{\partial\Phi_1}{\partial t}=\lambda\frac1r\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_1}{\partial r}\right),$$

$$\frac{\partial\Phi_2}{\partial t}=\lambda\frac1r\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_2}{\partial r}\right)+\frac{3}{8\Phi_0^4}\frac{\partial\Phi_1^2}{\partial t},$$

$$\frac{\partial\Phi_3}{\partial t}=\lambda\frac1r\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_3}{\partial r}\right)+\frac{1}{4\Phi_0^4}\frac{\partial}{\partial t}(\Phi_1\Phi_2)-\frac{7}{32\Phi_0^8}\frac{\partial\Phi_1^3}{\partial t},$$

. . . .

$$\lambda=\varkappa_{T0}\Phi_0^3.\tag{8}$$

This has to be solved subject to boundary and initial conditions for Φ_1, Φ_2, Φ_3, etc.

$$\Phi_1(r,0)=\Phi_2(r,0)=\Phi_3(r,0)=\ldots=0,$$

$$\Phi_1(\infty,t)=\Phi_2(\infty,t)=\Phi_3(\infty,t)=\ldots=0,$$

$$(r\,\partial\Phi_1/\partial r)_{r=r_c\to0}=-1.\tag{9}$$

Determination of $\Phi_1(r,t)$ amounts to solution of a linear differential equation of the type found in heat conduction [5], so we have

$$\Phi_1(r,t)=\mathrm{Ei}(-u)\quad u=\frac{r^2}{4\varkappa_{T0}t}\quad \mathrm{Ei}(-u)=-\int_u^\infty\frac{e^{-u}}{u}du.\tag{10}$$

In accordance with (8), we find that

$$\frac{3}{8\Phi_0^4}\frac{\partial}{\partial t}\Phi_1^2=\frac{6}{8\Phi_0^4t}\mathrm{Ei}(-u)e^{-u}.\tag{11}$$

Consequently,

$$\frac{\partial\Phi_2}{\partial t}=\lambda\frac1r\frac{\partial}{\partial r}\left(r\frac{\partial\Phi_2}{\partial r}\right)-\frac{3}{4\Phi_0^4t}\mathrm{Ei}(-u)e^{-u}.\tag{12}$$

We introduce the new variable $\xi=(u)^{1/2}=r/2(\varkappa_{T0}t)^{1/2}$, which gives (12) the form

$$\frac{d\Phi_2}{d\xi}\frac{\partial\xi}{\partial t}=\lambda\frac1r\frac{d\Phi_2}{d\xi}\frac{\partial\xi}{\partial r}+\lambda\frac{d^2\Phi_2}{d\xi^2}\left(\frac{\partial\xi}{\partial r}\right)^2+$$

$$+\lambda\frac{d\Phi_2}{d\xi}\frac{\partial^2\xi}{\partial r^2}\frac{3}{4\Phi_0^4t}\mathrm{Ei}(-u)e^{-u}.\tag{13}$$

We put

$$\frac{\partial\xi}{\partial t}=-\frac{r}{4\sqrt{\varkappa_{T0}t}\,t},\qquad\frac{\partial\xi}{\partial r}=\frac{1}{2\sqrt{\varkappa_{T0}t}},$$

which gives

$$\frac{d^2\Phi_2}{d\xi^2}+\frac{d\Phi_2}{d\xi}\left(2\xi\Phi_0^3+\frac1\xi\right)=\frac{3}{\Phi_0^7}\mathrm{Ei}(-u)e^{-u}.\tag{14}$$

As $\Phi_0 = 1$, we have from (14) that

$$\frac{d^2\Phi_2}{d\xi^2}+\frac{d\Phi_2}{d\xi}\left(2\xi+\frac1\xi\right)=3\mathrm{Ei}(-u)e^{-u}.\tag{15}$$

We get an inhomogeneous differential equation whose right part contains total derivatives; (15) may be solved by the method of varying the constants [6].

The fundamental system of (15) without the right part is $\mathrm{Ei}(-\xi^2)$, 1, so

$$\Phi_2=C_1(\xi)+C_2(\xi)\,\mathrm{Ei}(-\xi^2),\tag{16}$$

in which the variables $C_1(\xi)$ and $C_2(\xi)$ are defined from

$$\frac{dC_1}{d\xi}+\mathrm{Ei}(-\xi^2)\frac{dC_2}{d\xi}=0,$$

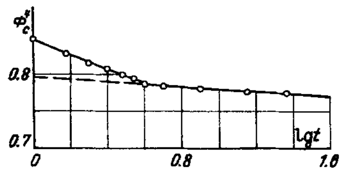

Fig. 1

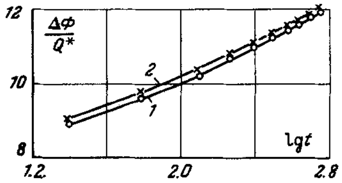

Fig. 2

$$\frac{d}{d\xi}\operatorname{Ei}(-\xi^2)\frac{dC_2}{d\xi} = 3\operatorname{Ei}(-\xi^2)e^{-\xi^2}. \qquad (17)\ \text{(cont'd)}$$

Integrating (17) and substituting into (16)

$$\Phi_2(r, t) = {}^3/_2 \operatorname{Ei}(-2u) - {}^3/_4\, e^{-u}\operatorname{Ei}(-u) - {}^3/_4\, \eta_2 \operatorname{Ei}(-u) + {}^3/_4\, \eta_1. \qquad (18)$$

The constants η_1 and η_2 are determined from (9), namely

$$\lim_{u\to\infty} \Phi_2 = 0, \quad \left(\xi \frac{\partial \Phi_2}{\partial \xi}\right)_{\xi=0} = 0,$$

and so $\eta_1 = 0$ and $\eta_2 = 1$. Then

$$\Phi_2(r, t) = {}^3/_2 \operatorname{Ei}(-2u) - {}^3/_4 e^{-u}\operatorname{Ei}(-u) - {}^3/_4 \operatorname{Ei}(-u). \qquad (19)$$

All subsequent approximations may be derived similarly.

ΔP, at	t, min	lg t	Φ_c^4	$\Phi_0^4 - \Phi_c^4$
0	0	—	1	0
9	0.5	—	0.9149	0.0851
16	1.0	0	0.8493	0.1507
18.5	1.5	0.18	0.8283	0.1717
20.0	2.0	0.30	0.8145	0.1855
21.0	2.5	0.40	0.8077	0.1923
22.0	3.0	0.48	0.7975	0.2025
22.5	3.5	0.54	0.7941	0.2059
23.0	4.0	0.60	0.7874	0.2126
23.5	5.0	0.70	0.7841	0.2159
23.8	8.0	0.90	0.7807	0.2193
24.3	14.0	1.15	0.7774	0.2226
24.8	23.0	1.36	0.7741	0.2259

We consider only the second approximation (error within permissible limits), and get the following solution to (2) for borehole startup:

$$\Phi_c^4(r_c, t) = 1 - \Omega \operatorname{Ei}(-u) + \Omega^2\, [{}^3/_2 \operatorname{Ei}(-2u) - {}^3/_4\, e^{-u}\operatorname{Ei}(-u) - {}^3/_4 \operatorname{Ei}(-u)]. \qquad (20)$$

Formula (20) requires laborious calculations, but for small u with $r = r_c$ it can be written as

$$\Phi_c^4(r_c, t) = 1 + \Omega \ln \frac{2.25\varkappa_{T0} t}{r_c^2} + \Omega^2\, 0.75 \ln \frac{4\varkappa_{T0} t}{r_c^2} - 2.783\,\Omega^2 =$$

$$= 1 + \Omega \ln \frac{2.25\varkappa_{T0}}{r_c^2} + 0.75\Omega^2 \ln \frac{4\varkappa_{T0}}{r_c^2} - 2.783\Omega^2 + (\Omega + 0.75\Omega^2)\ln t = B_1 + C_1 \ln t,$$

$$C_1 = \Omega(1 + 0.75\Omega),$$

$$B_1 = 1 + \Omega \ln \frac{2.25\varkappa_{T0}}{r_c^2} + 0.75\Omega^2 \ln \frac{4\varkappa_{T0}}{r_c^2} - 2.783\,\Omega^2, \qquad (21)$$

which shows that processing of pressure-reduction curves requires construction of a transformed graph in coordinates Φ_c^4 and log t, the slope of the straight-line part being determined. The resulting quadratic equation is solved to find Ω, which is used to determine k_{T0}. The accuracy is improved by introducing the fourth term in the expansion of (6), solving a cubic equation for log t, etc.

Figure 1 shows the transformed curve for borehole 160-5 (Malgobek-Voznesenskoe deposit), and this gives $k_{T_0} = 0.045$ darch.

Table 1 gives the data for the transformed curve, with the parameters $Q_v = 378$ m³/day, $\beta = 0.0025$ at^{-1}, $\mu = 0.306$ centipoise, h = 13 m, and k = 0.045 darcy.

This approximate result is compared with the self-modeling result in Fig. 2 for Q = 100 m³/day, $\beta = 0.005$ at^{-1}, $\mu = 1$ centipoise, h = 10 m, $k_{T0} = 0.01$ darcy, and $Q^* = 0.1$. The approximate solution is clearly very close to the exact solution.

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